

# MTH241 Fall 2024: Exam 02

Instructor: Dr. Ling Liang

Date: October 23, 2024

Time: 3:00pm - 3:50pm

Name:

UID:

Closed book, no calculator, show your work clearly.

1. (15pt) Mark True or False for each of the following statements. (**Grading:** 3pt each)
- (a) Let  $f(x, y)$  be a function in two variables. Then the graph of  $f$  is the same as the level surface of the function  $g(x, y, z) = f(x, y) - z$  with  $c = 0$ .  
**Your answer: True      False**
- (b) Let  $R$  be a set in the plane, then  $R$  must have an interior point.  
**Your answer: True      False**
- (c) Let  $f(x, y)$  be a function in two variables, then  $f_{xy}(x, y) = f_{yx}(x, y)$  always holds.  
**Your answer: True      False**
- (d) Let  $f$  have a relative extreme value at  $(x_0, y_0)$ , then  $f_x(x_0, y_0) = f_y(x_0, y_0) = 0$ .  
**Your answer: True      False**
- (e) Let  $f$  be a continuous function on a bounded set  $R$ , then  $f$  has both a maximum value and minimum value on  $R$ .  
**Your answer: True      False**
2. (35pt) Let  $f(x, y) = \sqrt{5 - 2x^2 - 2y^2}$ .
- (a) Compute the domain of  $f$  and express the level curve of  $f$  with  $c = 1$  using set notation. What is the shape of this level curve? (**Grading:** 3pt for working, 3pt for correct domain, 3pt for correct level curve, 1pt for correct shape.)

(b) Let  $\vec{a} = 3\vec{i} - 4\vec{j}$ , compute  $D_{\vec{a}}f(1, 1)$ . In what direction is  $f$  increasing most rapidly at  $(1, 1)$ ? (**Grading:** 4pt for working, 3pt for each correct answer.)

(c) Find a normal vector to the graph of  $f$  at  $(-1, -1, 1)$ , and find an equation of the plane tangent to the graph of  $f$  at  $(-1, -1, 1)$ . (**Grading:** 5pt for working, 5pt for correct normal vector, 5pt for correct tangent plane)

3. (15pt) Consider the following function:

$$w = f(x, y, z) = x + y + z, \quad x = e^{uv}, \quad y = \sin(u + v), \quad z = u^2 - v^2.$$

Compute the partial derivative  $\frac{\partial w}{\partial u}$  and  $\frac{\partial w}{\partial v}$  using the chain rule. (**Grading:** 5pt for working, 5pt for each partial derivative)

4. (10pt) Approximate  $\sqrt{2.98^2 + 4.04^2}$  using the tangent plane approximation. Simplify your final answer. (**Grading:** 5pt for working, 5pt for correct answer)

5. (25pt) Let  $f(x, y) = 2x^2 - 4x + 3y^2 + 1$ .

(a) Use the Second Partial Test to determine at which points  $f$  has relative extreme values (if any) and at which points  $f$  has saddle points (if any). (**Grading:** 5pt for working, 5pt for second partials test)

(b) Find all the extreme values of  $f$  on the disk  $x^2 + y^2 \leq 16$  by the Lagrange method. (**Grading:** 5pt for working, 10pt for extreme values)

Extra page: